Name:_____ Date:_____

Math 12 Honors: HW Section 3.4 Vieta's formula:

1. Given the roots, find the value of the coefficients without expanding:

a) $x^2+bx+c=(x-5)(x+7)$	b) $ax^2+bx+c=(2x+5)(3x-7)$
c) $x^3+bx^2+cx+d=(x-2)(x+3)(x-4)$	d) $ax^3+bx^2+cx+d=(x-3)^2(2x+1)$
e) $x^4+bx^3+cx^2+dx+e=(x-3)(x+3)(x-5)(x+5)$	f) $ax^4+bx^3+cx^2+dx+e=(2x+3)^2(x-4)(x+3)$

2. What is the sum of the squares of the roots of $4x^2+6x+2=0$?

3. If three roots of $x^4+Ax^2+Bx+C=0$ are -1, 2, and 3, then what is the value of $2C-AB$?

4. Given that the roots of $f(x) = x^3 - 4x^2 + 15x - 7$ are "a", "b", and "c", find the value of

   $i) a + b + c$ $ii) ab + bc + ac$ $iii) a \times b \times c$

5. Given that ""a", "b", "c" and "d" are the roots of $f(x) = 3x^4 + 2x^3 - 7x^2 + 9x - 10$, find the value of

   $abc + abd + acd + bcd$

6. Given that "a", "b", and "c" are the roots of $f(x) = 3x^3 - 4x^2 + 5x + 7$ find the value of the following:

   $i)\ a + b + c$ $ii) a^2 + b^2 + c^2$ $iii)\ \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

7. What is the sum of the reciprocals of the 5 solutions to: $5x^5 + 4x^4 - 3x^3 + 2x^2 + x - 1 = 2000$?

8. What is the arithmetic mean of the four solutions of $x^4 - 28x^3 + 265x^2 - 1006x + 1560 = 0$?

9. The polynomial equation $x^3 - 6x^2 + 5x - 1 = 0$ has three real roots: a, b, and c. Determine the value of $a^5 + b^5 + c^5$.

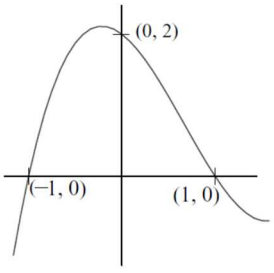
10. $f(x)=x^4+6x^3+ax^2-54x+c$ has four real roots: $r_1,\ r_2,\ r_3,\ and\ r_4$ such that $r_1+r_2=0$ and $r_4-r_3=4$. Find the values of coefficients "a" and "c". (AoPS)

11. Suppose $f(x)=x^3-5x^2+12x-19$ with roots "a", "b", and "c", what is the value of $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}$?

12. Suppose $r_1,\ r_2,\ r_3,\ .....\ r_{20}$ are the roots of $f(x)=x^{20}-19x+2$. What is the value of $r_1^{20}+r_2^{20}+r_3^{20}+....+r_{20}^{20}$?

13. What is the sum of all the coefficients (including the constant term) in the polynomial expansion of $(x-1)(x-2)(x-3)(x-4)\times.....\times(x-1996)(x-1997)$

14. Part of the graph of $y=f(x)=ax^3+bx^2+cx+d$ is shown. What is the value of "b"?



15. Challenge: Let "a", "b", and "c" be the roots of $x^3+3x^2-24x+1=0$. Suppose all three roots are real, show that $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}=0$ (AoPS)

(2003 AMC 10A #18) What is the sum of the reciprocals of the roots of the equation

$$\frac{2003}{2004}x+1+\frac{1}{x}=0?$$

(1991 MAΘ) Let r, s, t be the roots of $x^3-6x^2+5x-7=0$. Find

$$\frac{1}{r^2}+\frac{1}{s^2}+\frac{1}{t^2}.$$

(1983 AIME) What is the product of the real roots of the equation

$$x^2+18x+30=2\sqrt{x^2+18x+45}?$$